CS 250 **Midterm 1** **Oct 7, 2015**

Questions are worth 25 points each. You may use any results stated in class without having to re-prove them, except in problem 3 where you are explicitly asked to prove results already shown in class. Analyses should all be worst case unless stated otherwise.

1a) Insertion sort works by repeatedly adding a new element to the back of the list, and swapping it with the previous element until it is smaller than the previous element. Show that insertion sort is $O(n^2)$

1b) Explain why saying that insertion sort is $O(n^2)$ is not a good description of the worst case analysis of insertion sort.

1c) Complete the proof which gives a good description of the worst case running time of insertion sort.

2) Describe the efficient method of storing a set and performing UNION and FIND operations as was described in class. You must describe in particular how you perform the UNION operation and how you perform the FIND operation on this structure.

3) This question deals with edges of a graph which are not part of a spanning tree of a graph. Possible type of edges for an undirected graph are tree edges, back edges, or cross edges. Possible types of edges for a directed graph are tree edges, back edges, forward edges, or cross edges. For each of these 7 edge types, either show that the type of edge is possible with respect to a depth first search tree for a graph, or explain why the edge type cannot occur with respect to a depth first search tree.

4) A greedy algorithm for finding the shortest path from s to t in a complete weighted graph G works as follows. Start from s, and repeatedly select the shortest edge out of your current vertex to a vertex which has not yet been visited, until you reach t. Prove or disprove: this greedy algorithm always finds the shortest path from s to t.